An Innovative Approach to Minimizing Time of a Transportation Problem with Mixed Constraints

Farhana Rashid, Aminur Rahman Khan, Md. Sharif Uddin

Abstract--- In the literature, there are several methods for finding an initial basic feasible solution to time minimizing transportation problem with equality constraints. But a very unfortunate number of researchers proposed an algorithm for minimizing time in a transportation problem with mixed constraints. In this paper, a new approach is proposed for solving minimizing time in a transportation problem with mixed constraints. The proposed methods are easy to understand and to apply for finding an initial basic feasible solution to transportation problems happening in real-life situations.

Keywords: **Transportation problem (TP), Time Minimization Transportation Problem with Mixed Constraints (TMTP-MC), Initial Basic Feasible Solution (IBFS).**

1 Introduction

The transportation problem (TP) is a special class of linear programming problem, which deals with shipping commodities from sources to destinations. Transportation problem is to find the amount of commodity which should be transported from each source to each destination satisfying all the supply and demand limits of sources and destinations respectively so that the overall transporting cost is minimum which the objective of the TP. If the total supply and demand limits are equal, then the problem is said to be balance transportation problem with equality constraints.

The transportation problem was formalized by the French Mathematician Monge, [1].

Major advances were made in the field during World War II by the Russian Mathematician and Economist Leonid,Vitaliyevich Kantorovich [2]. The standard form of the transportation problem was first presented by Frank Lauren Hitchcock [3]. Efficient methods of solution were derived by T. C. Koopman [4],

Dantzig, G.B., [5] and then by Charnes A. et al. [6].

If the capacity of a source is extensively increased / decreased and the requirement of a destination is also extensively increased / decreased, then the overall transportation system converted to unequal

transportation problem with mixed constraints. The special type of transportation problem with mixed constraints was meticulously studied firstly by Brigden [7].

However now we describe some real situation about transportation. To sending disaster relief in aftermath of suddenly happened Hurricane, Earthquake , Tsunami or Flood. Any collaps, or a daring rescue mission, for any medical emergency such as Dengue, Chikungunya, Typhoid, or any other disease that spreading in epidemic form to tackle this situation sufficient medicine/diet or ambulance service could move as early as possible, weapons used in military operations, where in times of emergency, in the transportation of unpreserved goods such as fresh fruits and vegetables, etc. Same types of many other examples are exits around us where the speed of delivery is more important than the transportation cost. That's why Time Minimization Transportation Problem (TMTP) arises. In this paper we prefer to discussed Time Minimization Transportation Problem with Mixed Constraints. (TMTP-MC).

Time Minimization Transportation Problem (TMTP) with equality constraints was first addressed by Peter Ladislaw Hammer [8]. Garfinkel and Rao (Garfinkel R.S. and Rao M.R, [9] solved the time minimization transportation problem by introducing a sufficiently large cost on certain routes. Wlodzimierz Szwarc [10] compensation some of the deficiencies in Hammer's algorithm [8] and at the same time he also provided an extensive survey of the solution of TMTP with equality constraints. Extensive work has also been done on TMTP with equality constraints such as Bhatia [11], Sharma, J.K. and Swarup, K., [12,13], Seshan, C.R. and Tikekar, V.G., [14], Pandian and Natarajan [15] Sharma, A., Verma, V., Kaur, P. and Dahiya, K., [16], Uddin, M.S., [17], Nikolić, I., [18], Sharma, A. et al. [19], Uddin, M. M. et al. [20], Mollah Mesbahuddin Ahmed [21, 22], Khan, A.R. et. al. [23] have studied TMTPs and its variants. They introduced various algorithms for solving TMTP with equality constraints. From this literature review we see that a good amount of researches are available to obtain an optimal solution for the time minimizing transportation problem with equality constraints but the problem with mixed constraints is still touched by a very unfortunate number of researchers. The TMTP-MC is extensively useful in many real life situations where different sources and destinations require different limitations on supply and demand respectively according to their availability and

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requirement. Agarwal, S. and Sharma, S., [24] was first introduced Time Minimization Transportation Problem with Mixed Constraints (TMTP-MC). Also in 2018 Agarwal, S. and Sharma, S., [25] presented the method of finding IBFS of TMTP-MC is developed, with the same fundamental assumptions taken in their first method in 2014.

2 Time Minimization Transportation Problem with Mixed Constraints (TMTP-MC):

The structure of time minimization transportation problem with mixed constraints is similar to the classical transportation problem with mixed constraints, except that the unit transportation cost cij is replaced by the unit transportation time tij required to effect a complete shipment of the commodity from source i to destination j.
Therefore, the LPP model time minimization transportation problem with mixed constraints is

$$Minimize\ Z: \ \text{Max} \ \ t_{ij} \ \ |x_{ij} > 0$$

Subject to

Supply Constraints

$$\sum_{j \in T} x_{ij} = a_i, \quad i \in S_1$$
$$\sum_{j \in T} x_{ij} \geq a_i, \quad i \in S_2$$
$$\sum_{j \in T} x_{ij} \leq a_i, \quad i \in S_3$$

Demand Constraint

$$\sum_{i \in S} x_{ij} = b_j, \quad j \in T_1$$
$$\sum_{i \in S} x_{ij} \geq b_j, \quad j \in T_2$$
$$\sum_{i \in S} x_{ij} \leq b_j, \quad j \in T_3$$

And $x_{ij} \geq 0, \ \ i \in S, \ \ j \in T$

Where S_1, S_2, S_3 are partitioned from S, $i \in S$, $i = 1,2,3,\ldots\ldots..m$
Where T_1, T_2, T_3 are partitioned from T, $j \in T$, $j = 1, 2, 3,\ldots\ldots\ldots n$

We can also represent the above TMTP-MC by matrix form as follows:

Table 1: *Matrix Representation of TMTP-MC*

Factory	Warehouse				Supply
	1	2	......	n	
1	x_{11} t_{11}	x_{12} t_{12}	....	x_{1n} t_{1n}	$\geq / = / \leq a_1$
2	x_{21} t_{21}	x_{22} t_{22}	....	x_{2n} t_{2n}	$\geq / = / \leq a_2$
⋮	⋮	⋮	....	⋮	⋮
m	x_{m1} t_{m1}	x_{m2} t_{m2}		x_{mn} t_{mn}	$\geq / = / \leq a_m$
Demand	$\geq / = / \leq b_1$	$\geq / = / \leq b_2$	....	$\geq / = / \leq b_n$	

It assumed that:
1. The carries have sufficient capacity to carry goods from an origin to a destination in a single trip.
2. They start concurrently from their respective origins.

3 Theoretical Development of Proposed Method:

Step-1: Formulate the Time minimizing Transportation Problem with Mixed Constraints (TMTP-MC).
Step-2: Determine the average time for each row and column and put them in front of the row on the right and below the corresponding column respectively.
Step-:3: Select the cell containing minimum time corresponding to largest average time.
Step-4: Assign the supply and demand unit to the selected cell according to the following chart.

ai	≥			=			≤		
bj	=	≥	≤	=	≥	≤	=	≥	≤
Assign Unit	b	max(a,b)	b	min(a,b)	a	a	b	a	Min(a,b)

Step-5: Assign according to our chart to the cell with the lowest time in the row or column selected in step-3. If there is a tie it can be broken by selecting the cell where minimum time exist. Because our main objective is to find out minimum time. Allocate as our proposed chart and adjust supply and demand and cross off the necessary row/column.
Step-6: Calculate fresh average time for the remaining sub-matrix as in Step 2 and allocate and complete the procedure of Steps 3, 4 and 5. Continue the process until all rows and columns are satisfied.
Step-7: If all supply and demand exhausted, then go to step-9, otherwise step-8.
Step-8: Remaining (if necessary) on allocated supply /demand unit shift:
Where supply and demand constraints satisfies.
Choose in which cell is minimum for minimize time.

Otherwise, non-allocated unit should not allocate.
Step-9: Determine the largest time T0 corresponding to basic cells.

3 Numerical Illustration

For any medical emergency such as Dengue. Dhaka City Corporation Mayor (North and South) open four emergency medicine collection booth or warehouse in four corner in Dhaka city. They collect medicine from reputed three pharmaceutical company such as Square, Incepta and Aristopharma siuated outside in Dhaka. Square has production capacity of exactly 20 unit, Incepta has production capacity of at least 16 unit and Aristopharma has production capacity of at most 25 unit. Likewise collection booth-1 having capacity of demands at least 11 unit, booth-2 having capacity of demands at most 13 unit, booth-3 having capacity of demands at least 17 unit, booth-4 having capacity of demands exact 14 unit. Transportation time from factory to collection booth is t_{ij} which are given. From this type of problem we calculate the minimum transportation time with shipping unit using our proposed method
According to step-1, Formulate the given problem of (TMTP-MC).

		Collection Booth				Supply
		D1	D2	D3	D4	
Factory	S	1	6	3	5	=20
	I	7	3	1	6	≥16
	A	9	4	5	4	≤25
Demand		≥11	≤13	≥17	=14	

Then step by step completed the procedure and get the table below:

	D1	D2	D3	D4	Sup.	
S	20 1	6	3	5	=20	[3.8] -- --
I	7	2 3	17 1	6	≥16	[4.3] [3.3] [2]
A	9	11 4	5	14 4	≤25	[5.5] [4.3] [4.5]
Dem.	≥11	≤13	≥17	=14		
	[5.7] - -	[4.3] [3.5] [3.5]	[3] [3] [3]	[5] [5] -		

Therefore, the solution for the given problem is $x_{11} = 20$, $x_{22} = 2$, $x_{23} = 17$, $x_{32} = 11$, and $x_{34} = 14$

and the corresponding time of the cells are $t_{11} = 1$, $t_{22} = 3$, $t_{23} = 1$, $t_{32} = 4$ and $t_{34} = 4$.
Therefore, the total transportation time required

$$T_0 = \max\{t_{11}, t_{22}, t_{23}, t_{24}, t_{34}\}$$
$$= \max\{1, 3, 1, 4, 4\}$$
= 4

So, the total transported unit is 64 and required maximum time 4.

Conclusions

In this paper, we proposed an algorithm for minimizing time in a transportation problem with mixed constraints. In the proposed algorithm source and demands are represented by the combination of less equal, greater equal or equal to type constraints. To illustrate the proposed algorithm one numerical example is solved. Our proposed methods provide initial basic feasible solution in a simple and efficient manner, so the proposed approach is very easy to understand and to apply.

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